PATENT SPECIFICATION

697,864

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COMPLETE SPECIFICATION.

Improvements in or relating to Hair Springs.

We, S. SMITH & SONS (ENGLAND) LIMITED, a British Company, of Cricklewood Works, Cricklewood, London, N.W.2, do hereby declare the invention, for which we pray that a patent may be granted to us, and the method by which it is to be performed, to be particularly described in and by the following statement:—

The present invention relates to hair springs and is more particularly concerned with hair springs in the form of flat spirals.

Such springs are commonly used to provide a restoring or restraining torque on a rotatable shaft, the axis of the shaft being normal to the plane of the spring, the spring being anchored at its inner end to the shaft and being encastred at its outer end in a fixed part. It is frequently desirable that the restoring torque applied to the shaft should be, to a good approximation, proportional to the angle through which the shaft is turned from its equilibrium position. However, when the material of the spiral is uniform in cross-section and the encastering point for the outer end lies on the spiral, i.e. the final part is not deformed, or modified in any way, it is found that there is a departure from strict proportionality between the deflection and torque which has a sinusoidal component while the parts of the spring on the opposite side of the shaft from the fixed anchor point bunch together when the spring is wound up from its equilibrium position, those between anchor point and shaft retain substantially their original separation (apart from the decrease necessary to accommodate the additional turns); and this bunching is symtomatic of the component of error referred to above, subsequently referred to herein as "the sinusoidal component of error". An equivalent problem arises in horology and is discussed at length in a publication entitled "Memoire sur le spiral reglant" by Phillips (Paris, 1868). There it is shown that by deforming the final part of the outer turn of the spiral in a suitable manner the required proportionality, and consequent freedom from the bunching above referred to, can be obtained. However, these proposals entail shaping the final turn in such a manner that at least a part of it is a distance from the shaft which is less than the radius of the outer boundary of the spiral, i.e. a certain part must be in the form of an overcoil. Such an arrangement is not suitable when the shaft has to be rotated through an angle of the order of 360° or more from its equilibrium position, as under such conditions the spring tends to depart from its original plane, again causing departure from proportionality.

It is the object of the present invention to provide a hair spring in the form of a flat spiral which is such that when used in conjunction with a shaft in the manner described above the sinusoidal component of error is substantially eliminated.

According to the present invention a hair spring is formed from material substantially uniform in cross-section and spiral in form save for an outer terminal portion, which outer terminal portion is joined tangentially to the spiral portion, is of material of at least twice the stiffness of the major part of the spring, and is so shaped and arranged in the plane of the spiral that the sinusoidal component of error is substantially eliminated.

The outer terminal portion is preferably uniform in cross-section and in the form of a circular arc of extent not greater than 180°.

In a preferred form of the invention the material forming the outer terminal portion has twice the stiffness of the material forming the major part of the spring and is in the form of a substantially semi-circular arc with centre at the centre of the spiral.

The outer terminal portion is preferably provided with the desired elastic properties by modifying the outer terminal portion of a conventionally formed spring, as by plating, by joining to such portion an additional strip of material (to provide an increased cross-

section) or, in the process of forming the spring by rolling, by increasing the distance apart of the rollers for the formation of the outer terminal portion.

It will be appreciated by those skilled in the art that the requisite condition to be obeyed is that the reaction between the final terminal portion and the major part of the spring (and indeed across any section of the spring when the spring is in use should be a pure torque.

The design of a hair spring in accordance with the present invention in which the terminal portion is in the form of a circular arc will now be described with reference to the accompanying drawings of which :—

Figure 1 shows a plan view of the final portion of the major spiral part of a spring together with the terminal portion, in stressed and unstressed conditions;

Figure 2 shows a plan view of the terminal portion of a spring in stressed and unstressed conditions.

Figure 3 shows a plan view of a number of springs provided with terminal portions of differing radii and stiffnesses constructed in accordance with the invention, the main spiral portion being the same for all the springs.

The terminal portion of the hair spring in its unstressed condition is indicated at 1, and joins the main part of the spring indicated at 2, at p. Primed numbers and letters indicate these components and their letters in a stressed condition. The centre of the main part of the spring is indicated at 3, while the terminal portion of the spring is encastred, as indicated at 4. The centre of the arc formed by the terminal portion is indicated at 5.

If we suppose :—

The unstressed radius of the terminal portion is R.

The stressed radius of the terminal portion is $R + dR$.

The unstressed radius of the main part at p is r.

The stressed radius of the main part at p is $r + dr$.

The unstressed angular extent of the terminal portion is θ

The stressed angular extent of the terminal portion is $\theta + d\theta$.

The Young's modulus of the terminal portion is Ea.

The Young's modulus of the main part is Es.

The moment of inertia of cross-section of the terminal portion is Ia.

The moment of inertia of the main part is Is.

The bending moment in the stressed condition is M.

Then we have by the usual formula for bending, (and working throughout to the first order in dR, dr and $d\theta$) :—

$$\frac{M}{EaIa} = \frac{1}{R + dR} - \frac{1}{R}$$

so that,

$$dR = \frac{-R^2M}{EaIa}$$

and similarly

$$dr = \frac{-r^2M}{EsIs}$$

From the geometry of the arrangement (see Figure 2) :—

$$rd\theta = -(\theta - \sin\theta)\,dr$$

(measuring θ in radians and using the relationship $Rd\theta + \theta dR = 0$ as the length of the terminal portion is unchanged)

and

$$R = r\left(\frac{\theta}{\theta - \sin\theta}\right) \dots\dots\dots (1)$$

We also have :—

$$dr = (1 - \cos\theta)\,dR$$

so that

$$\frac{(1 - \cos\theta)\,R^2M}{EaIa} = \frac{r^2M}{EsIs}$$

and

$$\frac{Ea\,Ia}{Es\,Is} = \left(\frac{R}{r}\right)^2(1 - \cos\theta) \dots\dots\dots (2).$$

From the relations (1) and (2) it can be seen that if the major spiral part of the hair spring is specified, so that Es, Is and r are known, any one of the sets of quantities Ea, Ia, R, or θ can be specified, when the others are determined. It will usually be most convenient to specify θ. The correct value of Ea, Ia will be obtained by appropriate choice of material and cross-section. For example, if we make $\theta = 180°$.

$$R = r$$

and

$$Ea\,Ia = 2\,Es\,Is.$$

i.e. the terminal portion is in the form of a semi-circle centred at the centre of the spring and of twice the stiffness of the main part of the spring. It will not be possible to make θ greater than 180°, as this would entail making the terminal portion of smaller radius than the final turn of the main part of the spring leading to disadvantages referred to earlier. It will be seen that the conditions $Ea\,Ia \ngtr 2\,Es\,Is$, $\theta \nless 180°$, and $R \ngtr r$ are all equivalent.

Figure 3 shows in plan a number of hair

springs, constructed in accordance with the relations given. These all have the same main parts, and different terminal parts of form calculated in accordance with relations (1) and (2). The relevant quantities are given in the following table :—

Spring	θ	R/r	$\dfrac{Ea\ Ia}{Es\ Is}$
A	180°	1.00	2.00
B	165°	1.01	2.38
C	150°	1.22	2.83
D	135°	1.43	3.59
E	120°	1.71	4.38
F	105°	2.11	5.74
G	90°	2.74	7.52
H	75°	3.83	10.86
J	60°	5.76	16.68.

It will be seen that as θ is decreased, R/r increases fairly rapidly, as does also $Ea\ Ia/Es\ Is$. It will be appreciated that the stiffer the terminal part is made the smaller dR and $d\theta$ will become, so the larger will be the angle of movement of the main part of the spring which can be tolerated before errors of the kind referred to earlier become appreciable. In a practical case it will be necessary to choose a terminal portion of sufficient length and stiffness to give the required accuracy and not so long as to be inconvenient.

Springs constructed in accordance with the invention are particularly valuable for use as restraining springs in tachometers and the like having multi-turn pointers, as they can be conveniently made to accommodate a number of rotations without the appearance of substantial errors of the kind referred to earlier.

What we claim is :—

(1) A hair spring formed from material substantially uniform in cross-section and spiral in form save for an outer terminal portion, which outer terminal portion is joined tangentially to the end of the spiral portion, is of material of at least twice the stiffness of the major spiral part of the spring, and is so shaped and arranged in the plane of the spiral that the sinusoidal components of error (as earlier defined) is substantially eliminated.

(2) A hair spring as claimed in Claim 1, wherein the outer terminal portion is uniform in cross-section and is in the form of a circular arc of extent not greater than 180°.

(3) A hair spring as claimed in Claim 2, wherein the material forming the outer terminal portion has twice the stiffness of the material forming the major part of the spring and is in the form of a semi-circular arc with centre at the centre of the spiral.

(4) A hair spring as claimed in Claim 2, wherein the following relationships are obeyed :—

$$R = r\left(\frac{\theta}{\theta - \sin\theta}\right)$$

$$\frac{Ea\ Ia}{Es\ Is} = \left(\frac{R}{r}\right)^2 (1 - \cos\theta)$$

$$Ea\ Ia \ngtr 2\ Es\ Is.$$

where the various symbols have the meanings assigned to them in the text of the Specification.

(5) A hair spring as claimed in Claim 1, designed substantially in accordance with the procedure described herein and as shown in the accompanying drawings.

For the Applicants,
E. SWINBANK,
Chartered Patent Agent.

PROVISIONAL SPECIFICATION.

Improvements in or relating to Hair Springs.

We, S. SMITH & SONS (ENGLAND) LIMITED, a British Company, of Cricklewood Works, Cricklewood, London, N.W.2, do hereby declare this invention to be described in the following statement :—

The present invention relates to hair springs and is more particularly concerned with hair springs as the form of flat spirals.

Such a spring is commonly used to provide a restoring or restraining torque on a rotatable shaft, the axis of the shaft being normal to the plane of the spring, the spring being anchored at its inner end to the shaft and at its outer end to a fixed part. It is frequently desirable that the restoring torque applied to the shaft should be, to a good approximation, proportional to the angle through which the shaft is turned from its equilibrium position. However, when the material of the spiral is uniform in cross-section and the anchor point for the outer end lies on the spiral, i.e. the final part is not deformed, or modified in any way, it is found that there is a substantially sinusoidal departure from strict proportionality between the deflection and torque while the parts of the spring on the opposite side of the shaft from the fixed anchor point bunch together when the spring is wound up from its equilibrium position, those between anchor point and shaft retain substantially their original separation (apart from the decrease necessary to accommodate

the additional turns); and this bunching is symptomatic of the error referred to. An equivalent problem arises in horology and is discussed at length in a publication entitled "Memoire sur le spiral reglant" by Phillips (Paris, 1868). There it is shown that by deforming the final part of the outer turn of the spiral in a suitable manner the required proportionality, and consequent freedom from the bunching above referred to, can be obtained. However these proposals entail shaping the final turn in such a manner that at least a part of it is a distance from the shaft which is less than the radius of the outer boundary of the spiral, i.e. a certain part must be in the form of an overcoil. Such an arrangement is not suitable when the shaft has to be rotated through an angle of the order of 360° or more from its equilibrium position, as under such conditions the spring tends to depart from its original plane, again causing departure from proportionality.

It is the object of the present invention to provide a hair spring in the form of a flat spiral which is such that when used in conjunction with a shaft in the manner described above the restoring torque is substantially proportional to the angle through which the shaft is turned from its equilibrium position, the sinusoidal component of error referred to above being substantially eliminated.

According to the present invention a hair spring formed from material substantially uniform in cross-section and spiral in form save for an outer terminal portion, which outer terminal portion is joined tangentially to the spiral portion and is of material of at least twice the stiffness of the major part of the spring and is so shaped and arranged in the plane of the spiral that the sinusoidal component or error above referred to is substantially eliminated.

Most conveniently the outer terminal portion referred to is uniform in cross-section and in the form of a circular arc.

In the preferred form of the invention the outer terminal portion has twice the stiffness of the major part of the spring and is in the form of a substantially semi-circular arc with centre at the centre of the spiral.

The outer terminal portion may be formed in any convenient manner. It may be formed for example by plating the final portion of a conventionally formed spring, by joining to such final portion an additional strip of material of suitable cross-section and elastic properties or, in the process of forming the spring by rolling, by increasing the distance apart of the rollers for the formation of the outer terminal portion.

The precise length of the final terminal portion for any particular ratio of its stiffness to the stiffness of the major part of the spring may be calculated by the usual methods of calculation of spring properties. It will however, be found that the radius and length of the final terminal portion increase rapidly with increasing stiffness, rendering the use of a stiffness ratio much greater than two to one largely valueless.

A tachometer incorporating a hair spring according to the present invention will now be described.

The tachometer is of the conventional drag-cup type and comprises a framework in which are mounted, in suitable bearings, an input shaft carrying at one end and at right angles to itself a permanent magnet, together with an indicator shaft co-axial with the input shaft and carrying at one end a drag-cup adapted to fit over the poles of the magnet and at the other a pointer adapted to co-operate with a suitable scale, also mounted on the framework. A hair spring according to the invention is provided to restrain the indicator shaft against rotation. The hair spring is formed from a uniform strip of beryllium copper except for its outermost half turn which has brazed to it a further uniform strip of such a thickness that the stiffness of the final half turn is doubled. The inner end of the complete spring is encastred in a suitable projection formed on the indicator shaft while the outer end is encastred at a suitable point adjustable with respect to the framework. The plane of the complete spring is of course at right angles to the axis of the indicator shaft.

In operation rotation of the input shaft produces a torque upon the drag-cup proportional to the angular velocity of the input shaft. This produces a deflection of the indicator shaft and pointer substantially proportional to the angular velocity of the input shaft. This proportionality is maintained for several complete revolutions of the output shaft, and so a long effective scale length and high sensitivity are obtained. To resolve ambiguities a second pointer is provided, carried by a second indicator shaft geared to the first, and making less than one revolution for the complete range of the instrument. This second pointer also co-operates with a suitably calibrated scale.

For the Applicants,
E. SWINBANK,
Chartered Patent Agent.

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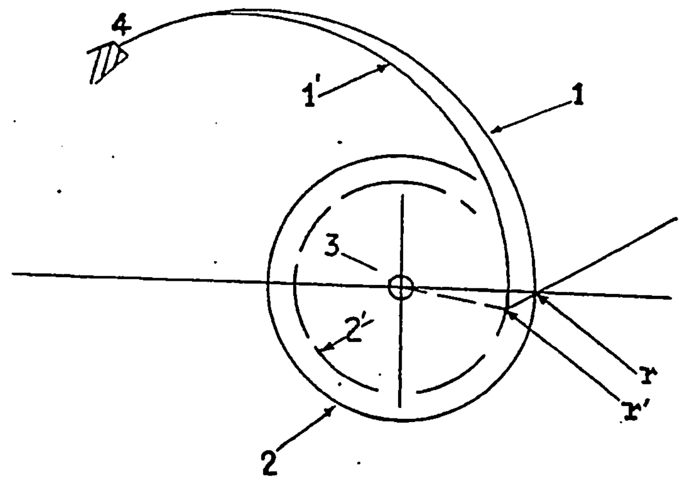
4
1'
1
3
2'
r
r'
2

FIG. 1

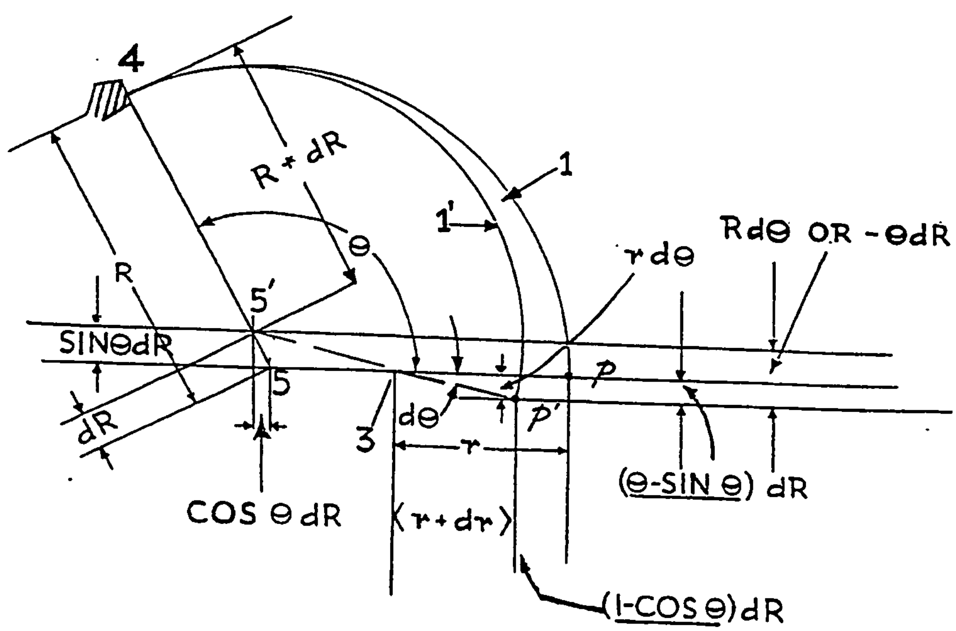
4
R + dR
1
1'
θ
R
Rdθ OR -θdR
rdθ
5'
SINθdR
5
p
dR
dθ
p'
3
r
(θ-SIN θ) dR
COS θ dR
(r+dr)
(1-COS θ)dR

FIG. 2

697,864 COMPLETE SPECIFICATION
2 SHEETS

This drawing is a reproduction of the Original on a reduced scale.
SHEETS 1 & 2

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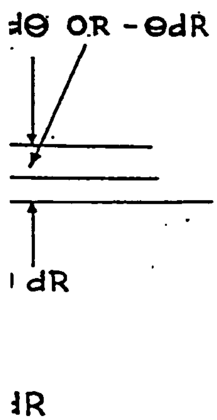

2

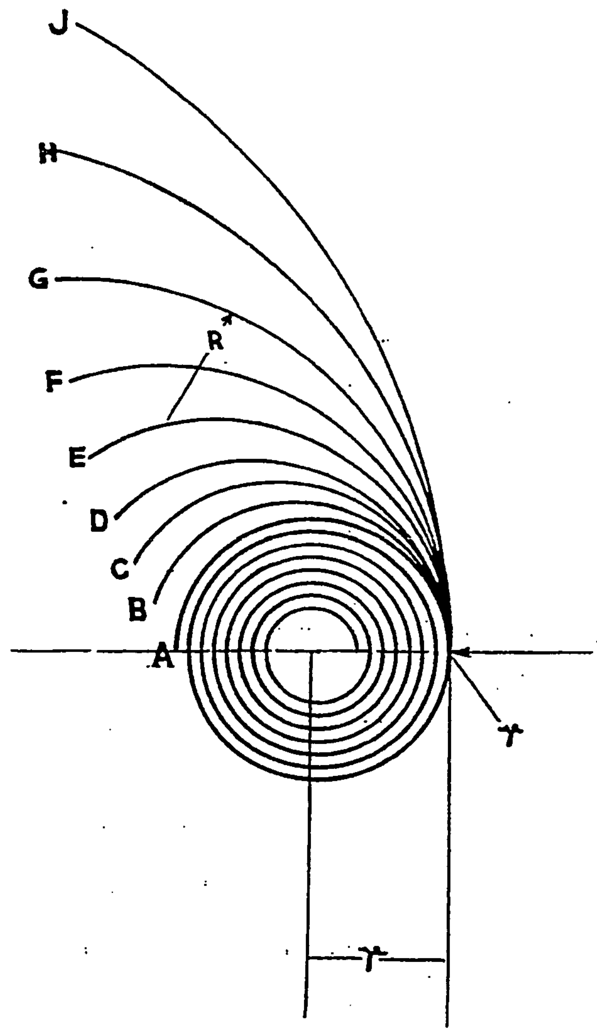

FIG 3

697,864 COMPLETE SPECIFICATION
2 SHEETS
This drawing is a reproduction of the Original on a reduced scale.
SHEETS 1 & 2

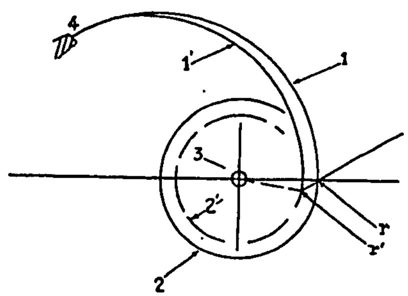

FIG. 1

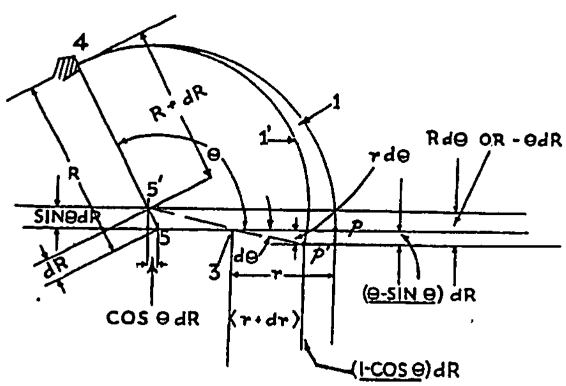

FIG. 2

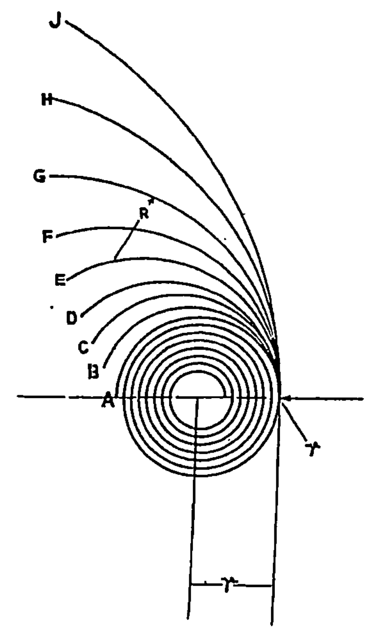

FIG 3